$SU(2)$ Kinetic Mixing Terms and Spontaneous Symmetry Breaking

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The non-abelian generalization of the Holdom model –*i.e.* a theory with two gauge fields coupled to the kinetic mixing term $g\ \mathrm{tr}\left(F_{\mu\nu}(A)F_{\mu\nu}(B)\right)$– is considered. Contrarily to the abelian case, the group structure $G\times G$ is explicitly broken to G. For $SU(2)$ this fact implies that the residual gauge symmetry as well as the Lorentz symmetry is spontaneusly broken. We show that this mechanism provides of masses for the involved particles. Also, the model presents instanton solutions with a redefined coupling constant.

I. INTRODUCTION

The search for new extensions of the Standard Model has stimulated the interest of the so called hidden sector of particles, which could interact very weakly with other known particles. In the context of string theory, the hidden sector appears naturally, predicting additional $U(1)$ factors [1, 2, 3, 5], with interesting phenomenological consequences [4].

This last idea has been explored in the abelian sector in [6], by considering two gauge fields interacting via a renormalizable gauge interaction by means of the following lagrangean

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}}, \tag{1}$$

where

$$\mathcal{L}_0 = \frac{1}{4}F^2_{\mu\nu}(A) + \frac{1}{4}F^2_{\mu\nu}(B), \tag{2}$$

and

$$\mathcal{L}_{\text{int}} = \frac{g}{2}F_{\mu\nu}(A)F_{\mu\nu}(B), \tag{3}$$

where g is a dimensionless coupling constant, A_μ, B_μ are abelian gauge fields and the strength tensor $F_{\mu\nu}$ is defined as usual, *i.e.* $F_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Equation (1) is the most general Lagrangian containing two gauge fields invariant under $U(1)\times U^{'}(1)$. Following Holdom [6] and others [7], (1) can be diagonalized very easily by using the transformation

$$B^{'}_\mu = B_\mu + gA_\mu. \tag{4}$$

and (1) becomes

$$\mathcal{L} = \frac{1}{4}(1-g^2)F^2_{\mu\nu}(A) + \frac{1}{4}F^2_{\mu\nu}(B^{'}), \tag{5}$$

where the electric charge now is redefined as

$$\tilde{e}^2 = \frac{1}{1-g^2}.$$

This last result was reached by Holdom [6] and recently this question has been revived in [7, 8] motivated by different reasons. However, to the best of our knowledge, there are no studies concerning to possible kinetic mixing in the non-abelian sector.

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Thus the goals of the present research are the following: firstly to generalize (1) to the non-abelian case and, secondly, to explore the physical meaning of the kinetic mixing terms. We will show below that this simple extension contains unsuspected properties such as, a) spontaneous symmetry breaking, and hence, appearance of Higgs bosons and massive gauge bosons, and b) vacuum instantons solutions.

The paper is organized as follows; in section II the free non-abelian theory containing two gauge fields is considered. In section III the kinetic mixing terms and its physical implications are discussed. Section IV is devoted to the spontaneous symmetry breaking phenomenon and generation of mass for gauge bosons which is present in the model. Section V contains a discussion on instantons and finally we conclude in section VI with some final remarks and outlook.

II. "FREE" NON-ABELIAN GAUGE FIELD THEORY

In order to expose our results, let us start by considering a model with two "free" gauge fields described by the Lagrangian

$$\mathcal{L}_0 = \frac{1}{4}\text{tr}\left(F^2_{\mu\nu}(A)\right) + \frac{1}{4}\text{tr}\left(F^2_{\mu\nu}(B)\right), \tag{6}$$

where $F_{\mu\nu}(A)$ and $F_{\mu\nu}(B)$ are the field strengths defined as usual

$$F_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu + e_A[A_\mu, A_\nu], \qquad F_{\mu\nu}(B) = \partial_\mu B_\nu - \partial_\nu B_\mu + e_B[B_\mu, B_\nu],$$

with $A \equiv A^a T^a$ and $B \equiv B^b S^b$, the gauge potential fields associated to two identical copies of $SU(2)$ and where T^a and S^a are the generators of $SU(2) \times SU(2)$ in a given representation. Thus, T and S are the generators of the first and the second copy of $SU(2)$ respectively.

The full symmetry group of the theory is $SU(2) \times SU(2)$, or in other words this system is invariant under the gauge transformations,

$$A_\mu \longrightarrow U_A^{-1} A_\mu U_A + e_A^{-1} U_A^{-1}\partial_\mu U_A, \tag{7}$$

and,

$$B_\mu \longrightarrow U_B^{-1} B_\mu U_B + e_B^{-1} U_B^{-1}\partial_\mu U_B, \tag{8}$$

where U_A and U_B are elements of $SU(2) \times SU(2)$, which act on the first and the second copy of $SU(2)$ respectively, and they are in general different.

Hence, one could define a unified connection $\mathcal{A}$ on $SU(2) \times SU(2)$, with normalized coupling constant $e = 1$, transforming as usual, *i.e.*

$$\mathcal{A}_\mu \longrightarrow \mathcal{U}^\dagger \mathcal{A}_\mu \mathcal{U} + \mathcal{U}\partial_\mu \mathcal{U}, \tag{9}$$

on the whole symmetry group.

Now, if we identify,

$$\mathcal{A}_\mu \equiv \alpha A_\mu \otimes I_s + \beta I_t \otimes B_\mu, \tag{10}$$

where α and β are constants, and $I_{t,s}$ stands for the identity element of the group in the representations T and S respectively. Then, using (7)-(10) one can check that the following consistency condition

$$\alpha = e_A, \qquad \beta = e_B,$$

must be fulfilled.

In order to find a Lagrangean for $\mathcal{A}$ it is convenient to rewrite the generators of the group as follows

$$\mathcal{T}^a = T^a \otimes I, \qquad \mathcal{S}^a = I \otimes S^a, \tag{11}$$

and then one can check that

$$\begin{aligned} \left[\mathcal{T}^a, \mathcal{T}^b\right] &= i\epsilon^{abc}\mathcal{T}^c, \\ \left[\mathcal{S}^a, \mathcal{S}^b\right] &= i\epsilon^{abc}\mathcal{S}^c, \\ \left[\mathcal{T}^a, \mathcal{S}^b\right] &= 0. \end{aligned} \tag{12}$$

However by defining the combinations

$$\mathcal{J}^a = \mathcal{T}^a + \mathcal{S}^a, \qquad \mathcal{K}^a = \mathcal{T}^a - \mathcal{S}^a, \tag{13}$$

which have the algebra

$$\begin{aligned} [\mathcal{J}^a, \mathcal{J}^b] &= i\epsilon^{abc}\mathcal{J}^c, \\ [\mathcal{J}^a, \mathcal{K}^b] &= i\epsilon^{abc}\mathcal{K}^c, \\ [\mathcal{K}^a, \mathcal{K}^b] &= i\epsilon^{abc}\mathcal{J}^c, \end{aligned} \tag{14}$$

which is –except by a sign in the last equation– the Lorentz group algebra. More precisely, this is the algebra for the rotations in $\mathbb{R}^4$, *i.e.*, $SO(4)$. With this in mind, we can extend the inner space in such a way that instead of having $a, b = 1, 2, 3$ we will have $\alpha, \beta = 1, 2, 3, 4$.

In this language, one can unify the two kind of generators as

$$\mathcal{M}^{\alpha\beta} = \left\{ \begin{array}{ll} \epsilon^{abc}\mathcal{J}^c & \text{if} \quad \alpha = a, \beta = b \\ \mathcal{K}^a & \text{if} \quad \alpha = 0, \beta = a \end{array} \right\} \tag{15}$$

and, therefore, the algebra (14) is summarized as

$$[\mathcal{M}^{\alpha\beta}, \mathcal{M}^{\gamma\delta}] = -i(\delta^{\alpha\gamma}\mathcal{M}^{\beta\delta} - \delta^{\beta\gamma}\mathcal{M}^{\alpha\delta} + \delta^{\beta\delta}\mathcal{M}^{\alpha\gamma} - \delta^{\alpha\delta}\mathcal{M}^{\beta\gamma})$$

We know the representations of this group, which can be thought of as the Lorentz group representations, but changing $\eta^{\mu\nu}$ for $-\delta^{\alpha\beta}$. In particular, the simplest nontrivial representation is given by the Dirac's matrices in the four-dimensional Euclidean space, where we use $i\gamma^0$ for Γ^0 and the other matrices as usual, *i.e.*,

$$\Gamma^\alpha = \begin{pmatrix} 0 & \sigma^\alpha \\ \bar{\sigma}^\alpha & 0 \end{pmatrix} \tag{16}$$

where $\sigma^0 = \bar{\sigma}^0 = iI_2$, and $\sigma^a = -\bar{\sigma}^a$ are the Pauli matrices for $a = 1, 2, 3$. Then, the representation for the generators are,

$$\mathcal{M}^{\alpha\beta} = \frac{i}{4}[\Gamma^\alpha, \Gamma^\beta]. \tag{17}$$

In particular,

$$\mathcal{J}^a = \frac{1}{2}\begin{pmatrix} \sigma^a & 0 \\ 0 & \sigma^a \end{pmatrix}, \qquad \mathcal{K}^a = \frac{1}{2}\begin{pmatrix} \sigma^a & 0 \\ 0 & -\sigma^a \end{pmatrix}, \tag{18}$$

and the analogous of γ^5 becomes

$$\Gamma^5 = -\Gamma^0\Gamma^1\Gamma^2\Gamma^3 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}. \tag{19}$$

This matrix has the property that $\{\Gamma^5, \Gamma^\alpha\} = 0$, and $\mathcal{K}^a = \Gamma^5\mathcal{J}^a$. This means that an element of $SU(2) \times SU(2)$ might be expressed like,

$$U = U_A \otimes U_B = \exp[i(\theta^a + \eta^a\Gamma^5)\mathcal{J}^a],$$

for matrices U_A and U_B in $SU(2)$ of the form,

$$U_A = e^{i\xi^a_A T^a}, \qquad U_B = e^{i\xi^a_B S^a},$$

where the parameters $\theta^a = (\xi^a_A + \xi^a_B)/2$ and $\eta^a = (\xi^a_A - \xi^a_B)/2$, are written in terms of the parameters ξ^a_A and ξ^a_B of the transformation U_A and U_B respectively. Hence, we can write,

$$\begin{aligned} F^a_{\mu\nu}(\mathcal{A}) &= \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu + [\mathcal{A}_\mu, \mathcal{A}_\nu] = e_A F^a_{\mu\nu}(A)\mathcal{T}^a + e_B F^a_{\mu\nu}(B)\mathcal{S}^a \\ &= \left(\frac{e_A F^a_{\mu\nu}(A) + e_B F^a_{\mu\nu}(B)}{2}\right)\mathcal{J}^a + \left(\frac{e_A F^a_{\mu\nu}(A) - e_B F^a_{\mu\nu}(B)}{2}\right)\mathcal{K}^a. \end{aligned}$$

So, in this representation,

$$F_{\mu\nu}(\mathcal{A}) = \frac{1}{2}\begin{pmatrix} e_A F^a_{\mu\nu}(A)\sigma^a & 0 \\ 0 & e_B F^a_{\mu\nu}(B)\sigma^a \end{pmatrix}. \tag{20}$$

Then with these facts in mind, one can see that the Lagrangean $\mathcal{L}_0$ can be written in terms of the curvature associated to $\mathcal{A}$. It reads,

$$\begin{aligned} \mathcal{L}_0 &= a\ \mathrm{tr} F^2_{\mu\nu}(\mathcal{A}) + a_5\ \mathrm{tr}\left(F_{\mu\nu}(\mathcal{A})\Gamma^5 F_{\mu\nu}(\mathcal{A})\right), & (21)\\ &= (a+a_5)\alpha^2 F^2_{\mu\nu}(A) + (a-a_5)\beta^2 F^2_{\mu\nu}(B), & (22) \end{aligned}$$

where the coefficients a and a_5 are defined as

$$\begin{aligned} a &= \frac{1}{8}\left(\frac{1}{e_A^2}+\frac{1}{e_B^2}\right), & (23)\\ a_5 &= \frac{1}{8}\left(\frac{1}{e_A^2}-\frac{1}{e_B^2}\right) & (24) \end{aligned}$$

The Lagrangean (21) and (22), of course, are equivalent to (6) with that choice for a and a_5. At this level one should note the following; firstly one can describe the "free" case using two formalisms, namely, in terms of $\mathcal{A}$ or in term of the pair (A, B) and both are equivalent descriptions. Secondly, these two descriptions can be physically interpreted following a simple formal analogy with fermions.

Indeed, let us assume a free massless fermion field which is described either by chiral fields or by Dirac ones. In the first case the Lagrangean for the chiral fields is

$$\mathcal{L} = \alpha_L\ \psi_L^\dagger\left(\partial_0 + \vec{\sigma}.\nabla\right)\psi_L + \alpha_R\ \psi_R^\dagger\ \left(\partial_0 - \vec{\sigma}.\nabla\right)\psi_R, \tag{25}$$

which is explicitly invariant under the chiral symmetry $SU(2)_L \times SU(2)_R$.

The second possibility is to use Dirac fields and the Lagrangean reads

$$\mathcal{L} = \frac{(\alpha_L+\alpha_R)}{2}\bar{\psi}\partial\!\!\!/\psi + \frac{(\alpha_L-\alpha_R)}{2}\bar{\psi}\gamma_5\partial\!\!\!/\psi, \tag{26}$$

which is invariant under the chiral and gauge symmetry transformations

$$\begin{aligned} \psi^{'}(x) &= e^{i\alpha_5\gamma_5}\psi(x), \qquad & \bar{\psi}^{'}(x) = \bar{\psi}^{'}(x)e^{i\alpha_5(x)\gamma_5}, \quad (27)\\ \psi^{'}(x) &= e^{i\alpha(x)}\psi(x), \qquad & \bar{\psi}^{'}(x) = \bar{\psi}^{'}(x)e^{-i\alpha(x)}, \quad (28) \end{aligned}$$

and, obviously, the Dirac version is also invariant under $SU(2)\times SU(2)$. So either versions are equivalent.

In our case at hands the fields A and B are the analogous of chiral fields ψ_L and ψ_R and –only in this sense, of course– (6) is a chiral description for two gauge fields. The field $\mathcal{A}$ is the analogous of a Dirac field with $\bar{\psi}\gamma_5\partial\!\!\!/\psi$ playing the role of $\mathrm{tr}\left(F(\mathcal{A})\Gamma^5F(\mathcal{A})\right)$.

Thus, an interesting point is the following; if a mass term in a fermionic theory breaks chiral symmetry, then, what is the analogous of a mass term in a gauge field theory as is discussed here?, and what are the physical implications?. We will answer these questions in the next section.

III. INCLUDING NON-ABELIAN KINETIC MIXING TERMS

Following the analogies discussed above we will consider the analogous of a "mass" term for a theory with two non-abelian gauge fields. This term should break partially the gauge symmetry in the same sense that $SU(2)\times SU(2) \sim SU(2)$ in the fermionic case when a term like $m(\psi_L^\dagger\psi_R + \psi_R^\dagger\psi_L)$ or $m\bar{\psi}\psi$ is added to (25) or (26).

In a field theory involving two gauge fields, however, this partial gauge symmetry breaking has important physical consequences as we will see below.

It is not difficult to see that this mass term must be

$$\mathcal{L}_I = \frac{g}{2}\mathrm{tr}\left(F_{\mu\nu}(A)F_{\mu\nu}(B)\right). \tag{29}$$

which, in turn, can be thought of as a non-abelian generalization of the Holdom and Okun model outlined in the introduction.

This term breaks partially the gauge symmetry $SU(2) \times SU(2)$ because if we perform the transformations (7)-(8), one finds

$$\mathcal{L}_I \longrightarrow \frac{g}{2} F^a_{\mu\nu}(A) \Lambda^{ab}(U_A^{-1}) \Lambda^{bc}(U_B) F^c_{\mu\nu}(B), \tag{30}$$

where Λ is in the adjoint representation of the $SU(2)$.

Thus, we must restrict ourselves to those transformations in $SU(2) \times SU(2)$ such that

$$U_A = U_B, \tag{31}$$

in order to keep them as symmetry transformations. Hence the residual symmetry becomes equivalent to $SU(2)$.

By writting the $\mathcal{A}$ field as,

$$\mathcal{A}_\mu = \mathcal{A}^{\alpha\beta} \mathcal{M}^{\alpha\beta} = Z^a_\mu \mathcal{J}^a + W^a_\mu \mathcal{K}^a, \tag{32}$$

where we have defined,

$$Z^a_\mu \equiv \frac{1}{2} \epsilon^{abc} \mathcal{A}^{bc} \equiv e_A A^a_\mu + e_B B^a_\mu, \tag{33}$$

and,

$$W^a_\mu \equiv \mathcal{A}^{0a} \equiv e_A A^a_\mu - e_B B^a_\mu. \tag{34}$$

one can see that under a gauge transformation that preserves the mass term, namely with $U_A = U_B \equiv U$, the fields Z and W transform as follows,

$$Z_\mu \to U^{-1} Z_\mu U + U^{-1} \partial_\mu U$$

and,

$$W_\mu \to U^{-1} W_\mu U$$

This says that Z is a gauge potential under the residual gauge symmetry, and W transforms in the adjoint representation.

In order to write down the mixing term in terms of the $\mathcal{A}$ field, and hence in terms of the Z and W fields, let us consider the the product $\Gamma^0 F_{\mu\nu}(\mathcal{A}) \Gamma^0$. It is actually easy to see that,

$$\Gamma^0 F_{\mu\nu}(\mathcal{A}) \Gamma^0 = -\frac{1}{2} \begin{pmatrix} e_B F^a_{\mu\nu}(B) \sigma^a & 0 \\ 0 & e_A F^a_{\mu\nu}(A) \sigma^a \end{pmatrix}.$$

And, then, the mixing term can be rewriten as,

$$\mathcal{L}_I = \frac{g}{2} F^a_{\mu\nu}(A) F^{a\mu\nu} = -\frac{g}{2} \mathrm{tr} \left(F_{\mu\nu}(\mathcal{A}) \Gamma^0 F^{\mu\nu}(\mathcal{A}) \Gamma^0 \right).$$

It is worthy to notice that the special role of the Γ^0 matrix of these term mimics the role of the γ^0 matrix in the fermionic analogy suggested above, and this fact justifies the "massive" name for this term.

The full Lagrangean of this "massive" model in terms of the connection $\mathcal{A}$ is, then,

$$\mathcal{L} = \frac{a}{4} \mathrm{tr} \left(F^2_{\mu\nu}(\mathcal{A}) \right) + \frac{a_5}{4} \left(F_{\mu\nu}(\mathcal{A}) \Gamma^5 F^{\mu\nu}(\mathcal{A}) \right) - \frac{g}{2} \mathrm{tr} \left(F_{\mu\nu}(\mathcal{A}) \Gamma^0 F^{\mu\nu}(\mathcal{A}) \Gamma^0 \right). \tag{35}$$

Following the fermionic analogy, $SU_5(2)$ invariance is broken in the same sense as chiral symmetry is broken in a massive fermionic field theory.

As we will see in the next section, some physical consequences of the model described by (35) can be understood more easily by expressing (35) in terms of Z^a_μ and W^a_μ. Then in terms of these fields the strength tensor is

$$F_{\mu\nu}(\mathcal{A}) = \left(F^a_{\mu\nu}(Z) + [W_\mu, W_\nu]^a \right) \mathcal{J}^a + \left((D_\mu W_\nu)^a - (D_\nu W_\mu)^a \right) \mathcal{K}^a \tag{36}$$

$$\Gamma^5 F_{\mu\nu}(\mathcal{A}) = \left((D_\mu W_\nu)^a - (D_\nu W_\mu)^a \right) \mathcal{J}^a + \left(F^a_{\mu\nu}(Z) + [W_\mu, W_\nu]^a \right) \mathcal{K}^a, \tag{37}$$

and,

$$-\Gamma^0 F_{\mu\nu}(\mathcal{A})\Gamma^0 = \left(F^a_{\mu\nu}(Z) + [W_\mu, W_\nu]^a\right)\mathcal{J}^a - \left((D_\mu W_\nu)^a - (D_\nu W_\mu)^a\right)\mathcal{K}^a \tag{38}$$

where the covariant derivative is $D_\mu W^a_\nu = \partial_\mu W^a_\nu + [Z_\mu, W_\nu]$. Thus,

$$\begin{aligned}\mathcal{L} &= \frac{1}{8e_A^2}\left\{\left(F^a_{\mu\nu}(Z) + [W_\mu, W_\nu]^a\right) + \left((D_\mu W_\nu) - (D_\nu W_\mu)\right)\right\}^2 \\ &+ \frac{1}{8e_B^2}\left\{\left(F^a_{\mu\nu}(Z) + [W_\mu, W_\nu]^a\right) - \left((D_\mu W_\nu) - (D_\nu W_\mu)\right)\right\}^2 \\ &+ \frac{g}{2}\left(F^a_{\mu\nu}(Z) + [W_\mu, W_\nu]^a\right)^2 - \frac{g}{2}\left((D_\mu W_\nu)^a - (D_\nu W_\mu)^a\right)^2,\end{aligned} \tag{39}$$

describes the full dynamics of a gauge field theory including the "massive" term (29).

IV. SPONTANEOUS SYMMETRY BREAKING AND MASS FOR Z^a_μ

The Lagrangean (39) contains several interesting physical properties as we will see in this section. The fact that we explicitly broke the full symmetry has a non trivial consequence; the residual $SU(2)$ symmetry is spontaneusly broken. As we pointed out at the end of the last section, Z^a_μ is a genuine gauge potential whereas W^a_μ is a vector field playing a role similar to the scalar one in the Higgs model.

To prove this statement, let us consider the lagrangean (39) with Z put to zero. Then, the potential energy for W, neglecting the spatial derivatives of W, is given, in the Euclidean space, by,

$$V[W] = \int d^4x\, \mathcal{V}(W) = \int d^4x\, \mathcal{L}(Z, W)|_{Z=\partial W=0}, \tag{40}$$

with $\mathcal{V}$

$$\mathcal{V}(W) = [W_\mu, W_\nu]^a[W^\mu, W^\nu]^a = \gamma\left[(\vec{W}^c\cdot\vec{W}^c)^2 - (\vec{W}^b\cdot\vec{W}^c)^2\right] \tag{41}$$

where the notation $\vec{W}^c\cdot\vec{W}^c$ means ${W^c}_\mu {W^c}_\mu$ and so on. Also, we have defined the constant

$$\gamma \equiv \left(\frac{1}{8e_A^2} + \frac{1}{8e_B^2} + \frac{g}{2}\right).$$

Now, we claim a nonvanishing vacuum expectation value for the field W, and then we redefine it in order to have expectation values on the vacuum equal to zero for the physical fields ω, *i.e.*,

$$W^a_\mu = v^a_\mu + \omega^a_\mu \tag{42}$$

To see the consistency of the above statement, we must see if it corresponds to an extremal point for the potential, *i.e.*, we must impose the condition,

$$\left.\frac{\partial\mathcal{V}}{\partial W^a_\mu}\right|_{W=v_0} = 0. \tag{43}$$

This expression produces the set of equations,

$$(\vec{v}^b\cdot\vec{v}^b)\vec{v}^a - (\vec{v}^a\cdot\vec{v}^b)\vec{v}^b = 0, \qquad a = 1, 2, 3. \tag{44}$$

It is easy to see –although not quite straightforward– that these equations have the general solution,

$$(v_0)^a_\mu = v\hat{\lambda}^a\hat{e}_\mu, \tag{45}$$

where v is an undetermined constant, $\hat{\lambda}^a$ and $\hat{e}_\mu$ are the components of unitary vectors in the inner and Euclidean spaces respectively, *i.e.*,

$$\sum_{a=1}^{3}(\hat{\lambda}^a)^2 = 1 = \sum_{\mu=1}^{4}(\hat{e}_\mu)^2.$$

So far, we have supposed that we are working in the Euclidean space, but, in short, we will see how it works for the Minkowski space.

The next question is whether or not these vacua are stable. In order to answer this question one must check the sign of the mass matrix, namely, to see that,

$$\frac{1}{2}\left.\frac{\partial^2\mathcal{V}}{\partial W^a_\mu \partial W^b_\nu}\right|_{W=v_0}\omega^a_\mu\omega^b_\nu \geq 0. \tag{46}$$

for any direction of ω. For our potential and the solutions (45), this yields,

$$\frac{1}{2}\left.\frac{\partial^2\mathcal{V}}{\partial W^a_\mu \partial W^b_\nu}\right|_{W=v_0} = 2\gamma v^2(\delta^{ab}-\hat{\lambda}^a\hat{\lambda}^b)(\delta_{\mu\nu}-\hat{e}_\mu\hat{e}_\nu). \tag{47}$$

However the condition (46) is satisfied only if,

$$g \geq -\left(\frac{1}{4e_A^2}+\frac{1}{4e_B^2}\right). \tag{48}$$

At this point it is worth noting that in the Minkowski space the only well defined matrix, *i.e.* positive or negative, is when $\hat{e}$ is a temporal-like vector, but because $\hat{e}^2=-1$ this matrix is negative defined. However, in the Minkowski space the potential energy has a different sign from the Euclidean potential energy, and hence, the Minkowski energy potential is positive defined for a temporal-like vector $\hat{e}$, and the answer does not change for the Minkowski space.

Thus assuming that (48) is hold, we obtain a nonvanishing mass for the W physical fields, *i.e.* we obtain,

$$m_W^2 = 2v^2\left(\frac{1}{4e_A^2}+\frac{1}{4e_B^2}+g\right), \tag{49}$$

for six of the twelve degrees of freedom associated to W^a_μ, and the other six massless excitations are Goldstone bosons. This last fact can be seen more clearly by choosing an orthonormal basis such that, $\vec{e}_4=\vec{e}$ and $\vec{e}_i$ with $i=1,2,3$, a set of three orthonormal vectors to $\vec{e}$ and to each other. Also, by choosing $\hat{\lambda}_3=\hat{\lambda}$ and $\hat{\lambda}_A$ with $A=1,2$. In this basis, the mass matrix is diagonal and it is written as,

$$\frac{1}{2}m_W^2\delta_{AB}\delta_{ij},$$

which says that only those components of W of the form,

$$W_{\text{massive}} = W_i^A\hat{\lambda}^A\otimes\hat{e}_i,$$

are massive.

Therefore although the mass of W^a_μ is hidden in the Lagrangean, the spontaneous symmetry breaking make it explicit.

Next step is to consider the coupling to the Z bosons. Following the standard arguments we can use the gauge freedom in order to remove two massless W bosons by choosing U such that

$$W_4^a = \vec{W}^a\cdot\hat{e} = U^{-1}(\hat{\lambda}^3+\omega)\mathcal{J}^3U$$

where U can be expressed as,

$$U = e^{i(\Psi^1\mathcal{J}^1+\Psi^2\mathcal{J}^2)}$$

for some suitable functions $\Psi^1(x)$ and $\Psi^2(x)$.

With this transformation the Z field changes as usual,

$$Z^a_\mu = U^{-1}\left[Z'_\mu+\partial_\mu\right]U. \tag{50}$$

The massless components W_4^1 and W_4^2 can be gauged out and, therefore, the mass matrix for this sector is,

$$\mathcal{L}_{mass}(Z',W') = \frac{1}{2}m_Z^2 Z_i'^A Z_j'^B\delta^{AB}\delta_{ij},$$

where

$$m_Z^2 = 2v^2\left(\frac{1}{4e_A^2}+\frac{1}{4e_B^2}-g\right). \tag{51}$$

These bosons will be stable if $m_Z^2 \geq 0$, therefore taking in account (48), the condition for having stable massive bosons is that,

$$|g| \leq \frac{1}{4}\left(\frac{1}{e_A^2}+\frac{1}{e_B^2}\right). \tag{52}$$

Otherwise, the vacuum is not spontaneusly broken and there would not be mass bosons.

The spectrum is then the following: one $U(1)$ massless gauge field Z_μ^3 (with two polarizations), two massive vector fields Z_i^1 and Z_i^2 (three polarizations each) and two massive vector fields W_i^1 and W_i^2 (three polarizations each). From this analysis also we get one massless scalar under rotation boson ω and one massless vector field W_i^3. However, If we couple gravity, the last massless vector fields W_i^3 and ω can be removed by a general coordinate transformation and, therefore, they do not not contribute to the spectrum in this sector.

One should note that in the case $g = 0$ the gauge symmetry $SU(2) \times SU(2)$ is recovered and, therefore, the mass terms are forbidden by the full gauge symmetry even though the mass expression is different from zero. Indeed, the shifting in the vacuum, (42), can be removed by a suitable gauge transformation.

V. INSTANTONS AND KINETIC MIXING TERMS

The model discussed above also have classical instanton solutions. Indeed, let us consider –as a warm-up exercise– the "free" case where the action in terms of two fields is

$$S = \frac{1}{4}\int d^4x\, F_{\mu\nu}^2(A) + \frac{1}{4}\int d^4x\, F_{\mu\nu}^2(A). \tag{53}$$

Then assuming self-duality conditions $F_{\mu\nu}(A) = \tilde{F}_{\mu\nu}(A)$ and $F_{\mu\nu}(B) = \tilde{F}_{\mu\nu}(B)$ one finds

$$S = 8\pi^2\left(\frac{n_A}{e_A^2}+\frac{n_B}{e_B^2}\right), \tag{54}$$

which is the standard instanton solution for two non-interacting gauge fields.

If we add the kinetic mixing terms (29) and we use (39) one finds that the relevant part of the action for the instanton calculations is

$$S = \left(\frac{e^2}{8e_A^2}+\frac{e^2}{8e_B^2}+\frac{g}{2}\right)\int d^4x F_{\mu\nu}^2(Z). \tag{55}$$

The other terms vanishes in $\mathbb{R}^4$ when $|x| \to \infty$ and, therefore, after to use the self-duality condition for $F_{\mu\nu}(Z)$ one finds

$$\begin{aligned} S &= \left(\frac{1}{8e_A^2}+\frac{1}{8e_B^2}+\frac{g}{2}\right)\int d^4x \tilde{F}_{\mu\nu}(Z)F_{\mu\nu}(Z), \\ &= \frac{8\pi^2}{\tilde{e}^2}n. \end{aligned} \tag{56}$$

where the redefined coupling constant is

$$\tilde{e}^2 = \left(\frac{1}{4e_A^2}+\frac{1}{4e_B^2}+2g\right)^{-1}. \tag{57}$$

Thus, in this $SU(2)$ case, the coupling constant also is redefined as in the Holdom model. The physical reasons, however, are completely different.

VI. FINAL REMARKS

In this paper a non-abelian extension of the Holdom model has been proposed. Contrarily to the abelian counterpart, the term $trF_{\mu\nu}(A)F_{\mu\nu}(B)$ breaks partially the $SU(2)\times SU(2)$ symmetry to $SU(2)$ implying interesting new physical properties. Among these new properties one can point out the following; the partial gauge symmetry breaking implies that only the combination (33) transforms as a gauge potential. Whereas (34) transforms as a matter field in the adjoint representation.

A careful analysis shows that the model proposed here is compatible with spontaneous symmetry breaking. Therefore, it provides mass for W_μ^a as in the standard Goldstone mechanism. Furthermore, the vector character of W_μ^a induces a spontaneous Lorentz symmetry breaking.

By coupling the Z_μ^a gauge fields to W_μ^a, some gauge bosons acquire mass as in the Higgs mechanism. Therefore, the model proposed here becomes equivalent to a vector Higgs mechanism. This fact together with the spontaneous Lorentz symmetry breaking seems like recent works on modified gravity [9] and composite model discussions [10, 11, 12, 13].

Another interesting aspect of this model is that the presence of an instanton solution suggests additional non-trivial properties of the vacuum. Furthermore, the coupling constat is effectively redefined $\tilde{e}$. This last fact could control possible divergences due to the explicit symmetry breaking, however, a proof of the renormalizability of the model proposed here is out of the scope of this paper.

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[1] K. R. Dienes, C. F. Kolda and J. March-Russel, *Nucl. Phys* **B492**, 104 (1997).
[2] T. R. Taylor, *Phys. Lett.* **B164**, 43 (1985).
[3] S. A. Abel and B. W. Schofield, *Nucl. Phys.* **B685**, 150 (2004).
[4] H. Murayama, Y. Nomura, D. Poland, *Phys. Rev.* **D77**, 015005 (2008); Z. Berezhiani, R. N. Mohapatra, *Phys.Rev.* **D52**, 6607 (1995); J. E. Kim, H. P. Nilles, *Phys.Lett.* **B263**, 79(1991); V. S. Berezinsky, A. Vilenkin, *Phys.Rev.* **D62**, 083512 (2000); S. A. Abel, M. D. Goodsell, J. Jaeckel, V. V. Khoze, A. Ringwald, *JHEP* **0807**, 124 (2008).
[5] L. B. Okun, *Sov.Phys.JETP* **56**, 502 (1982).
[6] B. Holdom, *Phys.Lett.* **B166**, 196 (1986).
[7] M. Ahlers, H. Gies, J. Jaeckel, J. Redondo, A. Ringwald, *Phys.Rev.* **D77**, 095001 (2008); S.N. Gninenko, e-Print:arXiv:0802.1315 [hep-ph]; D. Feldman, Z. Liu, P. Nath, arXiv:0810.5762.
[8] For a recent review on extensions of the standard model in the context of Holdom model see *e.g.*, P. Langacker, *"The Physics of Heavy Z-prime Gauge Bosons"* e-Print: arXiv:0801.1345 [hep-ph].
[9] Z. Berezhiani and O. V. Kancheli, e-Print: arXiv: 0808.3181; Z. Berezhiani, D. Comelli, F. Nesti and L. Pilo, *Phys. Rev. Lett.* **99**, 131101 (2007) and references therein.
[10] J. D. Bjorken, *Ann. of Phys.* **24**, 174 (1963), see also hep-th/011196.
[11] P. R. Phillips, *Phys. Rev.* **146**, 966 (1966).
[12] D. Atkatz, *Phys. Rev.* **17**, 1972 (1978).
[13] Y. Hosotani, *Phys. Lett.* **19B**, 332 (1993)